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Novel Method for Data Compression in Recursive INS Error Estimation

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In this work a new data compression method is presented which is applicable to systems whose observables are a linear combination of only a part of the system states. This characteristic is quite common in applied problems, especially in inertial navigation systems (INS). As a result, the formulation of the Kalman filter can be revised to yield a peculiar form for the covariance and state update which is the foundation of the new data compression method. The computations, according to this method, are divided into fast and slow rates. At the fast rate, which is determined by the availability of the measured data, a reduced-order Kalman filter is propagated and updated. The full-order system is propagated and updated only at a slow rate chosen by the designer. Utilizing the peculiar Kalman filter form of this case, the full-order system update is performed on the basis of the output of the reduced-order filter at each slow rate update time. Results of the application of this new data compression method to an INS are presented.

Introduction

CONSIDER the discrete time n-dimensional linear dynamic system whose state x_k at time k is generated by the difference equation

$$x_{k+1} = \phi_k x_k + B_k u_k + \Gamma_k w_k \tag{1}$$

with m-dimensional measurement

$$z_k = H_k x_k + v_k \tag{2}$$

The initial state x_0 of the system is assumed to be a random vector with

$$E\{x_0\} = x_0' \qquad \text{cov}\{x_0, x_0\} = P_0 \tag{3}$$

The control input $u_u \in R^r$ is assumed known (deterministic) for all k. The noises w_k and v_k are assumed to be uncorrelated white zero-mean random sequences with known covariances; that is, the q- and m-dimensional random vectors have the following second-order statistical properties for all k, $j=0,1,2,\ldots$:

$$E\{w_k\} = 0 \quad \text{cov}\{w_k, w_j\} = Q_k \delta_{kj} \quad Q_k \geq 0 \tag{4}$$

$$E\{v_k\} = 0 \quad \text{cov}\{v_k, v_j\} = R_k \delta_{kj} \quad R_k > 0 \tag{5}$$

$$\text{cov}\{w_k, v_j\} = 0$$

where δ_{kj} is the Kronecker delta defined by

$$\delta_{kj} = \begin{cases} 1 & \text{for } k=j \\ 0 & \text{for } k \neq j \end{cases}$$

In addition, it is assumed that

$$\text{cov}\{w_k, x_0\} = 0 \tag{6}$$

$$\text{cov}\{v_k, x_0\} = 0 \tag{7}$$

The matrices ϕ_k, B_k, Γ_k, and H_k are assumed to be known for all k and have compatible dimensions.

It is well known[1] that $\hat{x}_k$, the best linear estimate at time k of x_k, the system state at this time, where z_i, $i=0,1,\ldots,k$, are all given, may be found recursively by the Kalman filter algorithm

$$\hat{x}_k = \phi_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1} + K_k[z_k - H_k(\phi_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1})] \tag{8}$$

with the initial condition

$$\hat{x}_0 = x_0' \tag{9}$$

where the gain matrix K_k is given for all $k=0,1,2,\ldots$ by

$$K_k = P_{k/k-1} H_k^T [H_k P_{k/k-1} H_k^T + R_k]^{-1} \tag{10}$$

and where, in turn, the $n \times n$ non-negative definite matrix $P_{k/k-1}$ is the covariance of the estimation error

$$\tilde{x}_{k/k-1} = x_k - \hat{x}_{k/k-1} \tag{11}$$

at time k *before* the measurement z_k is processed; that is,

$$P_{k/k-1} \triangleq \text{cov}\{\tilde{x}_{k/k-1}, \tilde{x}_{k/k-1}\} \tag{12}$$

$P_{k/k-1}$ is obtained by

$$P_{k/k-1} = \phi_{k-1} P_{k-1} \phi_{k-1}^T + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \tag{13}$$

The covariance of the estimation error

$$\tilde{x}_k = x_k - \hat{x}_k \tag{14}$$

that is,

$$P_k \triangleq \text{cov}\{\tilde{x}_k, \tilde{x}_k\} \tag{15}$$

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is obtained by

$$P_k = (I - K_k H_k) P_{k/k-1} \tag{16}$$

In numerous cases where recursive estimation is applied to real problems, the rate at which the measurement data z_k are obtained surpasses by far the rate at which the computer can perform the estimation cycle; that is, solve Eqs. (8), (10), (13), and (16). This is, for example, a classical problem in inertial navigation systems (INS) where the vehicle position and/or velocity and/or the orientation of the INS stable platform are measured either continuously or at very short discrete intervals. The solutions to this problem which were proposed in the literature[2-12] are commonly known as "preprocessing" or "data compression" (not to be confused with data compression in communications theory, where this term has a different meaning[13]).

A solution to this problem which first comes to mind is the elimination of measurement data between the estimation instants whose frequency is determined by the capability of the computer. This, however, is an undesirable solution since a large amount of information may be lost. Another tempting solution is simple averaging of the measurement data. As a general solution this approach is deficient in that, from Eq. (2), the sequence of measurement vectors z_i is related to the respective states x_i at time i between two estimation instants $p-1$ and p (see Fig. 1) rather than to either x_{p-1} or x_p, which are estimated. To illustrate this point more vividly we turn to Fig. 2, where the scalar variable x changes linearly in time. Suppose that at $j=1$ we obtain two measurements of x, namely z_1 and z_2', which are spaced at an equal distance but in opposite directions from x_1; that is, $v_1 = -v_2$. A simple average will yield $\hat{x}_1 = \hat{x}_2$. If now, however, instead of obtaining the two simultaneous measurements z_1 and z_2' we obtain only z_1 and wait until the point $j=2$ to take the second measurement z_2, then, although the measurement error is still v_2 (as with z_2'), a simple averaging of z_1 and z_2 yields an $\hat{x}$ which is neither $\hat{x}_1$ nor $\hat{x}_2$.

This deficiency can be overcome using the dynamics of the system which for the general case are expressed by Eq. (1) to relate all the measurements between two estimation instants to the state of the system at either one of these instants. Schmidt, Weinberg, and Lukesh[5] made use of this approach assuming an autonomous system (i.e., $u_k = 0$ and $w_k = 0$), which enabled them to apply simple averaging to the measurement vectors. Joglekar and Powell[12] also investigated several averaging schemes where the dynamics of the system were used. Bar-Shalom[8,9] assumed an autonomous system with special dynamics and investigated the efficiency of linear and quadratic interpolators to compress the measured data. Dressler and Ross[6] proposed an entirely different kind of data compression scheme which disposes of averaging and according to which the state estimate $\hat{x}_k$ is computed every time a measurement is available but the gain matrix K_k is only updated every N iterations (see Fig. 1) and is held constant in the filter equations for the next N estimates before being updated anew. It should be noted that all the data compression methods mentioned here are approximate methods in that they do not yield the estimate which would have been obtained if a full Kalman estimate were performed at each measurement instant.

New Data Compression Method

We propose here a new method which holds for a special case which is often encountered in the real world, particularly in INS. The method is characterized by the application of the Kalman filter estimation algorithm to a reduced order system every time a new measurement is available, and a periodic estimation of the full state using the current estimate of a part of the reduced order state. We start the presentation with the following preliminary discussion.

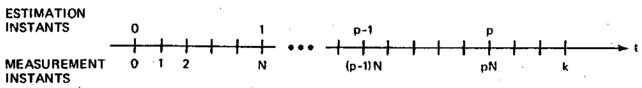

Fig. 1 Schematic description of the relative location of estimation instants with respect to measurement instants.

Renovation of the Covariance Matrix

In this section we will present a method which, given certain conditions, can be used to update the covariance matrix of the state estimation error.

Proposition 1: Given the linear system described in the introduction for which H_k is an $m \times n$ matrix of the form

$$H_k = [C_k \quad 0] \tag{17}$$

where C_k is an $m \times \nu$ matrix; then, sufficient conditions for computing the full P_k matrix are: the knowledge of $P_{k/k-1}$, the knowledge of ${}^{\nu\nu}P_k$ which is the upper left $\nu \times \nu$ part of P_k, and $|{}^{\nu\nu}P_{k/k-1}| \neq 0$.

Proof: Partition $P_{k/k-1}$ as follows

$$P_{k/k-1} = \begin{bmatrix} {}^{\nu\nu}P_{k/k-1} & {}^{\nu\eta}P_{k/k-1} \\ {}^{\nu\eta}P^T_{k/k-1} & {}^{\eta\eta}P_{k/k-1} \end{bmatrix} \tag{18}$$

where $\eta = n - \nu$. Then the application of Eqs. (10) and (16) with the H_k of Eq. (17) yields

$$P_k = \begin{bmatrix} {}^{\nu\nu}P_k & {}^{\nu\eta}P_k \\ {}^{\nu\eta}P^T_k & {}^{\eta\eta}P_k \end{bmatrix} \tag{19}$$

in which

$${}^{\nu\nu}P_k = [I - {}^{\nu\nu}P_{k/k-1}C_k^T(C_k{}^{\nu\nu}P_{k/k-1}C_k^T + R_k)^{-1}C_k]\,{}^{\nu\nu}P_{k/k-1} \tag{20}$$

$${}^{\nu\eta}P_k = [I - {}^{\nu\nu}P_{k/k-1}C_k^T(C_k{}^{\nu\nu}P_{k/k-1}C_k^T + R_k)^{-1}C_k]\,{}^{\nu\eta}P_{k/k-1} \tag{21}$$

$${}^{\eta\eta}P_k = {}^{\eta\eta}P_{k/k-1} - {}^{\nu\eta}P^T_{k/k-1}C_k^T(C_k{}^{\nu\nu}P_{k/k-1}C_k^T + R_k)^{-1} \times C_k{}^{\nu\eta}P_{k/k-1} \tag{22}$$

Noting that ${}^{\nu\nu}P_{k/k-1}$ is invertible, define

$$G_k \triangleq {}^{\nu\nu}P_k\,{}^{\nu\nu}P^{-1}_{k/k-1} \tag{23}$$

$$D_k \triangleq {}^{\nu\nu}P^{-1}_{k/k-1}(I - G_k) \tag{24}$$

Then from Eqs. (20) through (22) we obtain

$$P_k = \begin{bmatrix} {}^{\nu\nu}P_k & -G_k{}^{\nu\eta}P_{k/k-1} \\ {}^{\nu\eta}P^T_{k/k-1}G_k^T & {}^{\eta\eta}P_{k/k-1} - {}^{\nu\eta}P^T_{k/k-1}D_k{}^{\nu\eta}P_{k/k-1} \end{bmatrix} \tag{25}$$

which ends the proof since from Eqs. (23) through (25) the existence of the proposition is obvious.

Remark 1: From Eq. (20) it is evident that no part of $P_{k/k-1}$ other than ${}^{\nu\nu}P_{k/k-1}$ plays a role in the update of ${}^{\nu\nu}P_{k/k-1}$ itself.

Remark 2: If in some way (no matter how) we have obtained ${}^{\nu\nu}P_k$, then, assuming of course that $P_{k/k-1}$ is available and assuming that ${}^{\nu\nu}P_{k/k-1}$ is invertible, P_k can be obtained using the algorithm expressed by Eqs. (23) and (24). We call this algorithm *covariance renovation*.

The reason for adopting a new terminology is that if $P_{k/k-1}$ is given, P_k will be the Kalman-updated $P_{k/k-1}$ only when ${}^{\nu\nu}P_k$ is the true ${}^{\nu\nu}P_k$. If instead of the true ${}^{\nu\nu}P_k$ we compute an approximate matrix, then the renovation algorithm yields

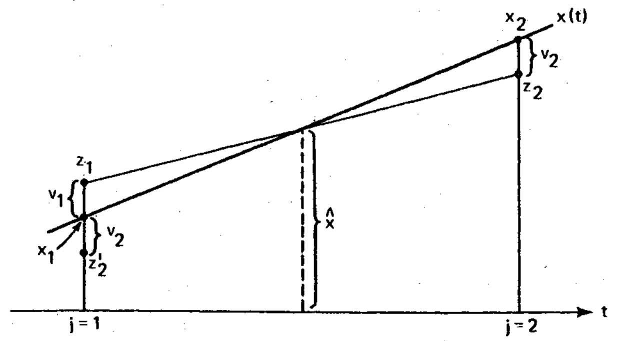

Fig. 2 Effect of simple measurement averaging on estimation of a scalar variable.

an approximate P_k which is not the Kalman-updated matrix. In addition, the term "update" will be used below to denote the process of obtaining an approximate value for ${}^{\nu\nu}P_k$.

A graphic illustration of the information flow during the renovation process is presented in Fig. 3. The light rectangles represent the partitioned $P_{k/k-1}$ matrix whereas the shaded ones represent the renovated matrix. The inputs to a rectangle represent the information necessary for computing the corresponding submatrix. The broken lines which enter the square denoted by ${}^{\nu\nu}P_k$ represent the information necessary to compute ${}^{\nu\nu}P_k$ using a Kalman update, in which case the shaded rectangles represent the submatrices which constitute P_k, the Kalman update of $P_{k/k-1}$.

Renovation of the State Vector

In this section we will present a method which, given the previous conditions, enables us to update the state estimate in a peculiar way.

Proposition 2: Given the conditions of Proposition 1, then sufficient conditions for computing the full state estimate $\hat{x}_k$ are the knowledge of $\hat{x}_{k-1}$ and of ${}^{\nu}\hat{x}_k$, the upper ν elements of $\hat{x}_k$.

Proof: Partition Eq. (8) as follows:

$${}^{\nu}\hat{x}_k = {}^{\nu n}\phi_{k-1}\hat{x}_{k-1} + {}^{\nu r}B_{k-1}u_{k-1} + {}^{\nu m}K_k\epsilon_k \tag{26}$$

$${}^{\eta}\hat{x}_k = {}^{\eta n}\phi_{k-1}\hat{x}_{k-1} + {}^{\eta r}B_{k-1}u_{k-1} + {}^{\eta m}K_k\epsilon_k \tag{27}$$

where ϵ_k is the innovation process.[14] Define

$$E_k \triangleq C_k^T(C_k\,{}^{\nu\nu}P_{k/k-1}C_k^T + R_k)^{-1} \tag{28}$$

Then, using Eq. (10) to compute K_k where H_k takes the special form expressed in Eq. (17), we obtain

$$K_k = \begin{bmatrix} {}^{\nu\nu}P_{k/k-1}E_k \\ {}^{\nu\eta}P^T_{k/k-1}E_k \end{bmatrix} \tag{29}$$

Then, from Eqs. (26) and (29),

$$E_k\epsilon_k = {}^{\nu\nu}P^{-1}_{k/k-1}[{}^{\nu}\hat{x}_k - {}^{\nu n}\phi_{k-1}\hat{x}_{k-1} - {}^{\nu r}B_{k-1}u_{k-1}] \tag{30}$$

From the lower part of K_k in Eq. (29) and from Eq. (30), Eq. (27) can be written as

$${}^{\eta}\hat{x}_k = {}^{\eta n}\phi_{k-1}\hat{x}_{k-1} + {}^{\eta r}B_{k-1}u_{k-1} + {}^{\nu\eta}P^T_{k/k-1}\,{}^{\nu\nu}P^{-1}_{k/k-1}({}^{\nu}\hat{x}_k - {}^{\nu n}\phi_{k-1}\hat{x}_{k-1} - {}^{\nu r}B_{k-1}u_{k-1}) \tag{31}$$

which ends the proof. Being derived from the Kalman filter, this result is not at all surprising since this is the form of the optimal linear least-square estimator.

Remark 3: If in some way (no matter how) we have obtained ${}^{\nu}\hat{x}_k$, then if $\hat{x}_{k-1}$ is available the rest of the latest state

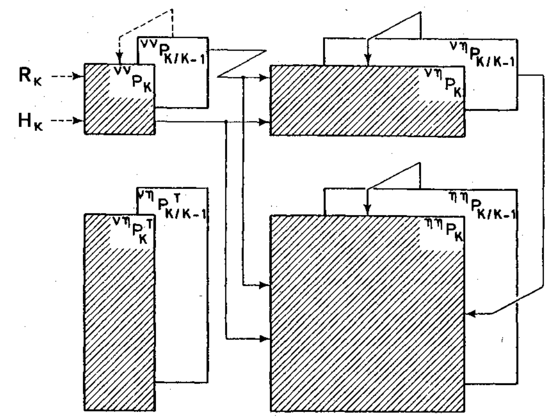

Fig. 3 Information flow during application of renovation algorithm.

estimate ${}^{\eta}\hat{x}_k$ can be obtained using Eq. (31). We call the algorithm of Eq. (31) *state renovation.*

An inspection of Eq. (31) yields the following interpretation of the mechanism by which the state renovation algorithm updates the rest of the state estimate. The difference between the best estimate of the first ν states given the last k measurements and that estimate given only $k-1$ measurements is weighed by ${}^{\nu\nu}P^{-1}_{k/k-1}$ and then by ${}^{\nu\eta}P^T_{k/k-1}$, the part of $P_{k/k-1}$ which correlates the ν states with the η states. The result is then added to the best estimate of the states given only $k-1$ measurements.

Outline of the New Data Compression Method

We assume that the measurement matrix in the case under consideration is of the form of Eq. (17). We construct a reduced order model which consists of at least those states which are transformed by C_k into the measurement vector; thus, since C_k is an $m\times\nu$ matrix, the lowest order of the reduced order model is ν. The purpose of the reduced order model is to reproduce the dynamic behavior of the system between the two estimation (renovation) instants $p-1$ and p (see Fig. 1) assuming the reduced order system is updated through a Kalman update every time a new measurement is available. Consequently, the number of states in the reduced order model must be at least the number of measured states. The main consideration for choosing additional states is their rate of change.

The reduced order model is used in the design of a reduced order Kalman filter which is then used to compute the estimate of the reduced order state every time a measurement is available; that is, the reduced order state is updated at the fast rate at which the measurements are available. Next, every N updates of the reduced order model the reduced order state is used to estimate the full order state vector. This is done in two steps: first the full order covariance matrix $P_{p/p-1}$ is renovated and then the full order state is renovated. The renovation of the covariance matrix is based on the assumptions that the propagated full-order covariance matrix is close to the correct $P_{p/p-1}$ and that ${}^{\nu\nu}P^*_p$, the upper left $\nu\times\nu$ submatrix of the reduced order matrix at the point $p=k$, is a good approximation of ${}^{\nu\nu}P_p$ of the full order covariance matrix should they have been updated at the fast rate; that is, we assume

$${}^{\nu\nu}P_p \approx {}^{\nu\nu}P^*_p \tag{32}$$

Then, following Eqs. (23) and (24), compute

$$G_p = {}^{\nu\nu}P^*_p\,{}^{\nu\nu}P^{-1}_{p/p-1} \tag{33}$$

$$D_p = {}^{\nu\nu}P^{-1}_{p/p-1}(I-G_p) \tag{34}$$

where $P_{p/p-1}$ is obtained by propagating the full order covariance matrix from the latest renovation point $p-1$ to the present point p. Then the full order matrix is renovated following Eq. (25):

$$P_p \approx \begin{bmatrix} {}^{\nu\nu}P_p^* & G_p {}^{\nu\eta}P_{p/p-1} \\ {}^{\nu\eta}P_{p/p-1}^T G_p^T & {}^{\eta\eta}P_{p/p-1} - {}^{\nu\eta}P_{p/p-1}^T D_p {}^{\nu\eta}P_{p/p-1} \end{bmatrix} \quad (35)$$

At the second step we renovate the state estimate. This renovation is based on the assumption that ${}^{\nu}\hat{x}_p^*$ (the vector which consists of the upper ν elements of the best estimate of the reduced order state vector at time p) is a good approximation of ${}^{\nu}\hat{x}_p$ (the corresponding part of the full order system) should it have been updated at the fast rate. In other words, we assume that

$${}^{\nu}\hat{x}_p \approx {}^{\nu}\hat{x}_p^* \quad (36)$$

Then, following Eq. (31), we obtain

$${}^{\eta}\hat{x}_p \approx {}^{\eta n}\phi_{p-1}\hat{x}_{p-1} + {}^{\eta r}B_{p-1}u_{p-1} + {}^{\nu\eta}P_{p/p-1}^T {}^{\nu\nu}P_{p/p-1}^{-1}({}^{\nu}\hat{x}_p^* - {}^{\nu n}\phi_{p-1}\hat{x}_{p-1} - {}^{\nu r}B_{p-1}u_{p-1}) \quad (37)$$

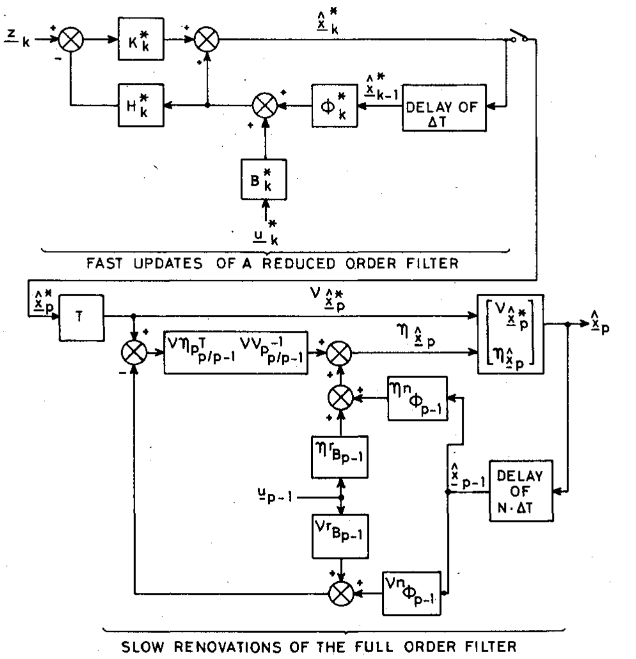

Fig. 4 Block diagram of multirate estimation process (data compression).Symbols with underbars correspond to boldface symbols in the text.

and the full state is the product of the augmentation of $\hat{x}_p^*$ and the result of Eq. (37). This algorithm is presented in Fig. 4 in a block diagram form. A time sequence of the same is shown in Fig. 5.

Note that ${}^{\nu}\hat{x}_p^*$ contains the information added by the measurements to the reduced order filter. While the renovation algorithm does not require a measurement noise matrix, such a matrix is implied by the algorithm.

The initial reduced order covariance matrix as well as the reduced order covariance matrix after each renovation are extracted from the full order covariance matrix (see Fig. 5). Another possibility is that of performing a maximum likelihood estimate on the reduced order system, which means assigning a very large value to the reduced order covariance matrix initially and after each renovation.

Finally, it should be noted that the covariance renovation in Eq. (35) is based on the simple Kalman filter covariance update equation given in Eq. (16). It is well known that a computationally superior form of Eq. (16) is the following Joseph algorithm:[15]

$$P_k = (I-K_kH_k)P_{k/k-1}(I-K_kH_k)^T + K_kR_kK_k^T \quad (38)$$

The covariance renovation algorithm which is based on Eq. (38) is given in the Appendix.

Application to INS

In inertial navigation systems the general linear dynamic error equation consists of three position states, three velocity states, three orientation states, and in certain cases one or more damping states. This error equation is driven by a nonwhite forcing function vector. The latter requires the addition of shaping filters and thus up to a threefold to fourfold increase of the initial number of states. Moreover, when transfer alignment takes place, that is, when this INS serves as a reference for the erection and alignment of another INS, the states of the latter INS, as well as other states, are added to the states of the first INS. The total number of states may therefore exceed one hundred. On the other hand, the number of states involved in external measurements is always less than ten (unless the measuring system error model is augmented with the previous states, in which case some of these added states are also involved in the measurements). Typically, the number of measured states is less than seven. This small ratio between measured and unmeasured states is the factor which makes the new data compression method very attractive in INS.

According to the scheme presented here, a reduced order filter which contains at least those ν states involved in the measurements has to be designed. The accuracy to which this filter reproduces the dynamics of the ν states is one of the major factors that determine the accuracy of the data compression scheme. This is so because the deviation of ${}^{\nu\nu}P_{k/k-1}^*$ from ${}^{\nu\nu}P_{k/k-1}$ and of ${}^{\nu}\hat{x}_k^*$ from ${}^{\nu}\hat{x}_k$ are significant factors that determine the accuracy of the scheme. These deviations are

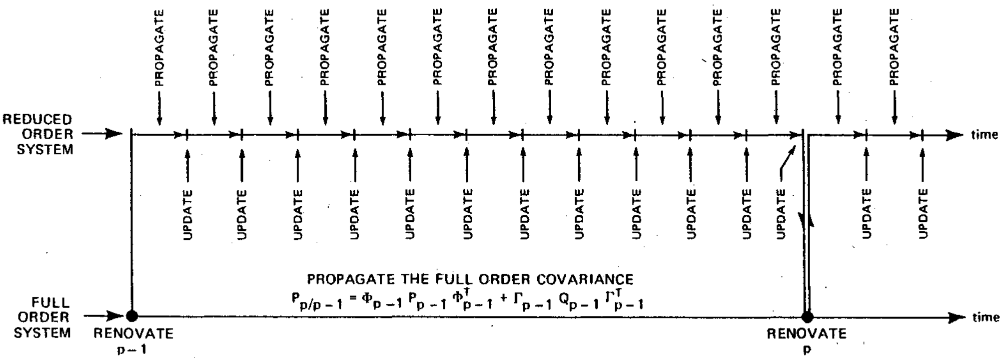

Fig. 5 Time sequence of multirate estimation process (data compression).

determined by the initial values, the propagation accuracy, and the update accuracy of the reduced order filter. The initial values are identical whether a full Kalman filter or the data compression schemes are used. As stated in Remark 2, the updates of the first ν states are done identically whether they are done in the full system or in the reduced order system. Thus the only step which may be performed differently is the propagation, and the propagation of the reduced order filter depends on the particular choice of its dynamic model.

The particular INS for which the new data compression method was applied consists of 34 states which describe the INS error propagation in the two horizontal channels. The vertical channel was not included since, as is well known, this channel is practically decoupled from the horizontal channels. The measurements were the two horizontal position and two horizontal velocity components; hence, ν was 4 and in the state vector of the reduced order filter we included the corresponding two position and two velocity error states. In addition, the state vector included the three INS attitude error states and two velocity damping states; hence, the order of the reduced order filter was 9. The dynamics matrix of the reduced order filter ${}^{99}F_k^*$ was simply ${}^{99}F_k$; that is, it was the upper left 9×9 part of F_k, the full order dynamic matrix. Similarly ${}^{99}Q_k^*$, the covariance of the reduced order filter noise, was ${}^{99}Q_k$, which is zero; hence, the dynamics of the reduced order filter were those of an autonomous system. The initial covariance matrix ${}^{99}P_{p-1}^*$ of the reduced order filter for the pth data compression interval, was ${}^{99}P_{p-1}$, the upper left 9×9 part of the renovated full order matrix.

Note that while the minimum number of states which have to be included in the reduced order filter is four, we chose, for the sake of better simulating the dynamics of these four states, to add five more states since, as was mentioned earlier, the accuracy in fitting the reduced order filter dynamics to the true dynamics of the states involved in the measurement is an important factor which affects this data compression scheme. Note that under the conditions discussed here a reduced order filter whose states are merely the four measured states assumes that the velocity errors are constant through the data compression interval, a case which was treated by Bar-Shalom.[8,9] Our choice to include the attitude error states assumes that while the INS tilt and azimuth errors are nonzero, they do not vary during the data compression interval. This simple-minded reduced order filter was accurate enough for our purposes and was very easy and economic to implement. One can, of course, design a more accurate and thus a more elaborate filter if need arises in particular cases.

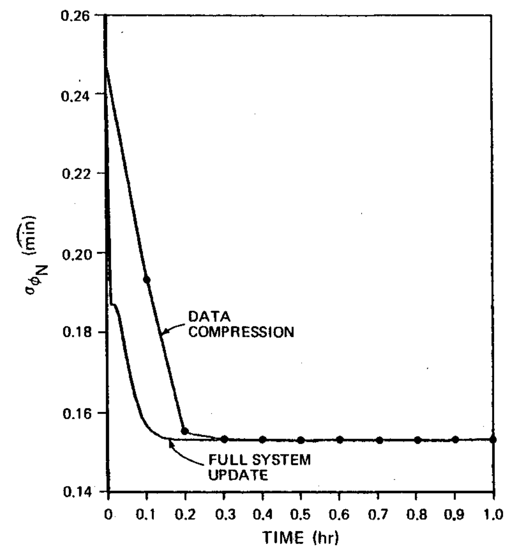

Fig. 6 Covariance simulation results of new data compression method.

Results of covariance simulations which present the effect of the new data compression method are shown in Fig. 6. The variable plotted in Fig. 6 is the standard deviation of the estimation error of ϕ_N, the platform tilt about the north axis. We purposely chose to present the effect of this data compression method on a state not included in the four measured states since the standard deviation of this state is obtained by renovation. Two plots are shown in Fig. 6. The lower plot is that of σ_{ϕ_N} when the system is updated every time new data are available, which in this example is every 36 seconds. The upper plot is that of σ_{ϕ_N} when the new data compression method is used to process all the available measurements. The dots are the values of σ_{ϕ_N} after renovation and they are computed by the data compression algorithm. The straight lines between the dots are generated by the plotter and are not computed by the data compression algorithm.

Conclusion

In this paper we presented a new data compression method. The new method is based on the fact that, when the measurements are a subset of the state, the Kalman filter algorithm takes a peculiar form. The method consists of fast Kalman updates of a reduced order filter and slow approximate updates (renovation) of the full order system. The accuracy of the method strongly depends on the ability of the reduced order system to simulate the dynamics of the states which are involved in the measurements.

The assumption that the measurements are a linear combination of only a part of the system states, an assumption on which this method is based, is seemingly a very special feature; however, this is usually the case when a Kalman filter is applied to real problems and it is especially true in the application of the Kalman filter to INS. In fact, this is precisely the reason why the Kalman filter has been used in a wide range of INS applications. An example in which the new data compression method was applied to an INS was given and covariance simulation results were presented. When the INS is a high quality system and the measurements contain very little noise, data compression is not useful since most of the information obtained by the measurements is superfluous; however, in cases where data compression is useful, the new method is a good working algorithm.

Appendix

The purpose of this appendix is to present the covariance renovation algorithm based on the computationally advantageous Joseph algorithm of Eq. (38). The following algorithm is the result of the expansion of Eqs. (10) and (38) with the H_k of Eq. (17). Given $P_{k/k-1}$ and ${}^{\nu\nu}P_k$, compute

$$G_k = {}^{\nu\nu}P_k\,{}^{\nu\nu}P_{k/k-1}^{-1} \tag{A1}$$

$$D_k = {}^{\nu\nu}P_{k/k-1}^{-1}(I-G_k) \tag{A2}$$

$$E_k = D_k G_k \tag{A3}$$

Then compute the matrices U_k and V_k as follows

$$U_k = \begin{bmatrix} {}^{\nu\nu}P_k & G_k^2\,{}^{\nu\eta}P_{k/k-1} \\ {}^{\nu\eta}P_{k/k-1}^T (G_k^T)^2 & {}^{\eta\eta}P_{k/k-1} - {}^{\nu\eta}P_{k/k-1}^T (I+G_k^T) D_k\,{}^{\nu\eta}P_{k/k-1} \end{bmatrix} \tag{A4}$$

$$V_k = \begin{bmatrix} 0 & {}^{\nu\nu}P_{k/k-1} E_k\,{}^{\nu\eta}P_{k/k-1} \\ {}^{\nu\eta}P_{k/k-1}^T E_k\,{}^{\nu\nu}P_{k/k-1} & {}^{\nu\eta}P_{k/k-1}^T E_k\,{}^{\nu\eta}P_{k/k-1} \end{bmatrix} \tag{A5}$$

Finally

$$P_k = U_k + V_k \tag{A6}$$

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